Coupled Mode Analysis of Multi-conductor Transmission Lines Including Backward Coupling

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Abstract---**The non-orthogonal coupled mode theory is extended to the analysis of multi-conductor transmission lines by including the backward wave coupling. Scattering parameters can be obtained directly from the coupled mode analysis. The eigenmode field and current are calculated using the finite difference time domain (FDTD) method, which can provide a wide band solution through a single simulation. Numerical results for the case of two identical coupled lines are presented and compared with those from ADS Momentum.**

I. INTRODUCTION

Coupled mode theory has been widely applied to microwave, optoelectronic and fiber optic problems due to its physical intuitiveness and mathematical simplicity [1]. When considering the coupling among transmission lines, it is convenient to choose the superposition of the modes of the individual waveguides as the trial solution of the N-guide system. The modes of different waveguides are non-orthogonal in the sense of power. This has been noticed and the non-orthogonal coupled mode theory for parallel dielectric waveguides was developed [1]-[4]. The consideration of this non-orthogonality gives a more accurate description of the problem, especially when the coupling is strong. K.Yasumoto [5] extended the theory to the analysis of multilayered and multiconductor transmission lines by the use of a generalized reciprocity relation. This theory was applied to the evaluation of line parameters of coupled microstrip lines [6].

In the parallel dielectric waveguide case, the backward coupling is negligibly small and can be ignored for most practical applications [7]. The backward coupling in multiconductor transmission lines can no longer be ignored because in many cases, the backward coupling is even stronger than the forward one. In this paper, we will extend the theory developed in [5] by considering the backward coupling.

In coupled mode equations, the coupling coefficients are obtained by overlap integrals between the eigenmode fields and currents of individual transmission lines. Generally, the eigenmode can be obtained by any numerical methods. Reference [5] used Galerkin's moment method in the spectral domain. In this paper, we use the finite difference time domain (FDTD) method, which can provide broadband information of eigenmodes in a single simulation.

We apply this generalized non-orthogonal coupled mode theory to the analysis of two coupled microstrip lines. Scattering parameters can be obtained directly from the coupled mode analysis including counter-propagating waves. The results are compared with those obtained from the full-wave commercial software.

II. NON-ORTHOGONAL COUPLED MODE FORMULATION

Consider N lossless coupled microstrip lines located in M layers, as shown in Fig.1.

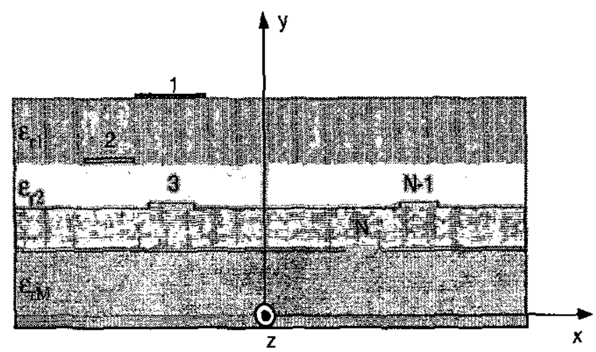

Fig. 1. Cross section of N coupled microstrip lines in M layer dielectric substrate.

We start from the generalized Lorentz reciprocity relation for a z-translational invariant system that is derived in [5] as:

$$\frac{\partial}{\partial z}\int_S (\boldsymbol{E}_1 \times \boldsymbol{H}_2 - \boldsymbol{E}_2 \times \boldsymbol{H}_1)\cdot \boldsymbol{i}_z dS = j\omega \int_S (\varepsilon_1 - \varepsilon_2)\boldsymbol{E}_1 \cdot \boldsymbol{E}_2 dS + \int_S \boldsymbol{E}_2 \cdot \boldsymbol{J}_1 dS - \int_S \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dS \tag{1}$$

where $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$ are the electromagnetic fields produced by a current source $\boldsymbol{J}_1$ located in one dielectric medium with permittivity ε_1 and permeability μ_0, and $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$ are the electromagnetic fields produced by a current source $\boldsymbol{J}_2$ located in another dielectric medium with permittivity ε_2 and permeability μ_0.

We choose $\boldsymbol{E}_1$, $\boldsymbol{H}_1$ and $\boldsymbol{J}_1$ to be the fields and currents of N parallel microstrip lines, which can be approximated by the linear superposition of eigenmodes of the individual lines:

$$\boldsymbol{E}_1(x,y,z) = \sum_{\nu=1}^{N}\left(a_\nu^+(z)\boldsymbol{e}_\nu^+(x,y) + a_\nu^-(z)\boldsymbol{e}_\nu^-(x,y)\right) \tag{2a}$$

$$\boldsymbol{H}_1(x,y,z) = \sum_{\nu=1}^{N}\left(a_\nu^+(z)\boldsymbol{h}_\nu^+(x,y) + a_\nu^-(z)\boldsymbol{h}_\nu^-(x,y)\right) \tag{2b}$$

$$\boldsymbol{J}_1(x,y,z) = \sum_{\nu=1}^{N}\left(a_\nu^+(z)\boldsymbol{j}_\nu^+(x,y) + a_\nu^-(z)\boldsymbol{j}_\nu^-(x,y)\right), \tag{2c}$$

where $a_\nu(z)$ are unknown amplitudes of the modes, $\boldsymbol{e}_\nu(x,y)$, $\boldsymbol{h}_\nu(x,y)$ and $\boldsymbol{j}_\nu(x,y)$ are the eigenmode x- and y-dependant functions of individual microstrip line, and the superscripts + and – represent the modes propagating in $+z$ and $-z$ directions.

We further choose $\boldsymbol{E}_2$, $\boldsymbol{H}_2$ and $\boldsymbol{J}_2$ to be the backward propagating eigenmode of the μth individual microstrip line, which have the form of:

$$\boldsymbol{E}_2(x,y,z) = \boldsymbol{e}_\mu^-(x,y)e^{j\beta_\mu z} \tag{3a}$$

$$\boldsymbol{H}_2(x,y,z) = \boldsymbol{h}_\mu^-(x,y)e^{j\beta_\mu z} \tag{3b}$$

$$\boldsymbol{J}_2(x,y,z) = \boldsymbol{j}_\mu^-(x,y)e^{j\beta_\mu z} \tag{3c}$$

Substituting (2) and (3) into (1), and noticing that in this case,

$$\int_S \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 dS = 0, \tag{4}$$

also using the relation of fields and currents of guided modes in the opposite directions,

$$\boldsymbol{e}_\nu^\pm(x,y) = \boldsymbol{e}_{\nu t}(x,y) \pm \boldsymbol{i}_z e_{\nu z}(x,y) \tag{5a}$$

$$\boldsymbol{h}_\nu^\pm(x,y) = \pm\boldsymbol{h}_{\nu t}(x,y) + \boldsymbol{i}_z h_{\nu z}(x,y) \tag{5b}$$

$$\boldsymbol{j}_\nu^\pm(x,y) = \boldsymbol{j}_{\nu t}(x,y) \pm \boldsymbol{i}_z j_{\nu z}(x,y), \tag{5c}$$

we can arrive at the following equations for the mode amplitudes:

$$\sum_\nu\left[\frac{1}{2}(N_{\nu\mu}+N_{\mu\nu})\left(\frac{\partial}{\partial z}a_\nu^+ + j\beta_\mu a_\nu^+\right) + \frac{1}{2}(N_{\nu\mu}-N_{\mu\nu})\left(\frac{\partial}{\partial z}a_\nu^- + j\beta_\mu a_\nu^-\right)\right] = -j\sum_\nu\left[(K_{\mu\nu}^t - K_{\mu\nu}^z + L_{\nu\mu}^t - L_{\nu\mu}^z)a_\nu^+ + (K_{\mu\nu}^t + K_{\mu\nu}^z + L_{\nu\mu}^t + L_{\nu\mu}^z)a_\nu^-\right] \tag{6a}$$

where

$$N_{\nu\mu} = \frac{1}{2}\int_S (\boldsymbol{e}_{\nu t} \times \boldsymbol{h}_{\mu t})\cdot \boldsymbol{i}_z dS \tag{6b}$$

$$K_{\mu\nu}^t = -\frac{j}{4}\int_S (\boldsymbol{e}_{\mu t}\cdot \boldsymbol{j}_{\nu t})dS \tag{6c}$$

$$K_{\mu\nu}^z = -\frac{j}{4}\int_S (e_{\mu z}\cdot j_{\nu z})dS \tag{6d}$$

$$L_{\nu\mu}^t = \frac{\omega}{4}\int_S \Delta\varepsilon(\boldsymbol{e}_{\nu t}\cdot \boldsymbol{e}_{\mu t})dS \tag{6e}$$

$$L_{\nu\mu}^z = \frac{\omega}{4}\int_S \Delta\varepsilon(e_{\nu z}\cdot e_{\mu z})dS \tag{6f}$$

$$\Delta\varepsilon = \varepsilon_1 - \varepsilon_2 \tag{6g}$$

Following a similar procedure, choosing $\boldsymbol{E}_2$, $\boldsymbol{H}_2$ and $\boldsymbol{J}_2$ to be the forward propagating eigenmode of the μth individual line, and using the parameters above, we can obtain another equation:

$$\sum_\nu\left[\frac{1}{2}(N_{\nu\mu}-N_{\mu\nu})\left(\frac{\partial}{\partial z}a_\nu^+ - j\beta_\mu a_\nu^+\right) + \frac{1}{2}(N_{\nu\mu}+N_{\mu\nu})\left(\frac{\partial}{\partial z}a_\nu^- - j\beta_\mu a_\nu^-\right)\right] = j\sum_\nu\left[(K_{\mu\nu}^t + K_{\mu\nu}^z + L_{\nu\mu}^t + L_{\nu\mu}^z)a_\nu^+ + (K_{\mu\nu}^t - K_{\mu\nu}^z + L_{\nu\mu}^t - L_{\nu\mu}^z)a_\nu^-\right] \tag{7}$$

Equations (6) and (7) are the coupled mode equations for multi-conductor transmission lines after taking counter-propagating waves into consideration. If we choose the two systems having the same dielectric medium ($\Delta\varepsilon$=0), the coefficient $L_{\nu\mu}$ will be zero.

If we only consider co-propagating modes, equations (6) and (7) reduce to:

$$\sum_\nu\left[\frac{1}{2}(N_{\nu\mu}+N_{\mu\nu})\left(\frac{\partial}{\partial z}a_\nu^+ + j\beta_\mu a_\nu^+\right)\right] = -j\sum_\nu\left[(K_{\mu\nu}^t - K_{\mu\nu}^z)a_\nu^+\right], \tag{8}$$

which has the same form as that derived in [5].

III. FDTD Analysis of Eigenmodes

The FDTD method has already been applied in the extraction of transmission line parameters [8], including those of multiple coupled lines [9]. In order to compute the 2N port parameter matrix from the FDTD data, each port should be excited individually [9]. When the configuration, for example the distances of the coupled lines, changes, the above procedure has to be repeated. In the coupled mode analysis, only the eigenmode field and current of each line need to be calculated. The results can be reused if the structure is composed of the microstrip lines having the same parameters.

We use a high order FDTD method [10] to obtain the eigenmode field and current of a single microstrip line. A Gaussian pulse with an appropriate bandwidth both in the time domain and the space domain is used to stimulate the microstrip. Perfectly matched layer (PML) absorbing boundary conditions are used at the top and two sides, as well as at the input side to minimize reflections. After propagating for a certain distance, only the dominant mode is left because higher order modes are evanescent in the frequency range that are of interest. Each point of the time domain field in the cross-section is transformed into the frequency domain by Fourier transform. The propagation constant of the mode is acquired by comparing the phases at two locations along the z-direction.

An example of the Ey component of eigenfield for a microstrip line calculated by the FDTD method is shown in Fig.2. The parameters of the microstrip line are: dielectric constant of the substrate $\varepsilon_r = 9.8$, height of the substrate h=0.2mm, width of the microstrip w =0.2mm.

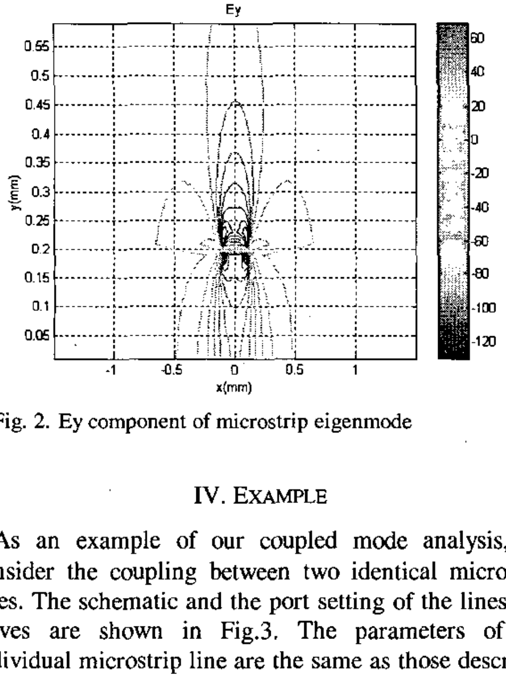

Fig. 2. Ey component of microstrip eigenmode

IV. EXAMPLE

As an example of our coupled mode analysis, we consider the coupling between two identical microstrip lines. The schematic and the port setting of the lines and waves are shown in Fig.3. The parameters of the individual microstrip line are the same as those described in Section III. The coupling length L=6mm and the separation s=0.2mm. The metal strips are assumed to be perfectly conducting and infinitely thin.

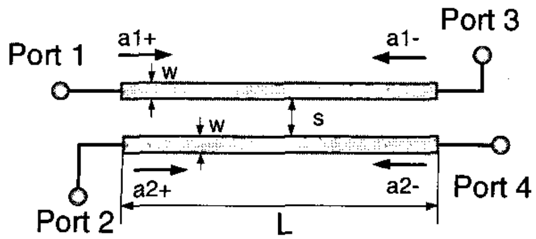

Fig. 3 Symmetric coupled microstrip lines

In the case of two identical lines, four waves are involved in the problem. The coupled mode equation has a simple form as:

$$\begin{bmatrix} 1 & N & 0 & 0 \\ N & 1 & 0 & 0 \\ 0 & 0 & 1 & N \\ 0 & 0 & N & 1 \end{bmatrix} \frac{d}{dz} \begin{bmatrix} a_1^+ \\ a_2^+ \\ a_1^- \\ a_2^- \end{bmatrix} = -j \begin{bmatrix} \beta_0 & \beta_0 N + \kappa^+ & 0 & \kappa^- \\ \beta_0 N + \kappa^+ & \beta_0 & \kappa^- & 0 \\ 0 & -\kappa^- & -\beta_0 & -\beta_0 N - \kappa^+ \\ -\kappa^- & 0 & -\beta_0 N - \kappa^+ & -\beta_0 \end{bmatrix} \begin{bmatrix} a_1^+ \\ a_2^+ \\ a_1^- \\ a_2^- \end{bmatrix} \quad (9)$$

where $N = (N_{12} + N_{21})/2$, $\kappa^{\pm} = K_{12}^t \mp K_{12}^z = K_{21}^t \mp K_{21}^z$, $\beta_0 = \beta_1 = \beta_2$ is the propagation constant of the individual line.

The eigenvalues of this equation, which are corresponding to the propagation constants of the even- and odd-mode, are:

$$\beta_e^{\pm} = \pm\sqrt{\left(\beta_0 + \frac{\kappa^+ - \kappa^-}{1+N}\right)\left(\beta_0 + \frac{\kappa^+ + \kappa^-}{1+N}\right)} \quad (10a)$$

$$\beta_o^{\pm} = \pm\sqrt{\left(\beta_0 - \frac{\kappa^+ - \kappa^-}{1-N}\right)\left(\beta_0 - \frac{\kappa^+ + \kappa^-}{1-N}\right)} \quad (10b)$$

From the previous theory [5], which only considers the co-propagating waves, these parameters have the form of

$$\beta_e = \beta_0 + \frac{\kappa^+}{1+N} \quad (11a)$$

$$\beta_e = \beta_0 - \frac{\kappa^+}{1-N}, \quad (11b)$$

which are the first-order approximation of equation (10).

If we ignore the effect of non-orthogonality among the modes of individual lines, the parameter N in equation (11) will not exist.

In the frame of coupled mode theory, the three-dimensional full-wave problem is converted to one-dimensional boundary value problem. The load terminations can be included in the analysis as a form of boundary value condition (BVC). The characteristic impedances of the individual transmission line are needed to determine these BVC and they can be obtained when calculating the eigenmode of these waveguides. Here, for simplicity, we assume that these ports are terminated with the characteristics impedance of the individual lines. The BVC of such a setting is:

$$a_1^+(0) = 1,\ a_2^+(0) = 0,\ a_1^-(L) = 0,\ a_2^-(L) = 0 \quad (12)$$

We can define the scattering parameters with these four waves:

$$S_{11} = \frac{a_1^-(0)}{a_1^+(0)},\ S_{21} = \frac{a_2^-(0)}{a_1^+(0)} \quad (13a)$$

$$S_{31} = \frac{a_1^+(L)}{a_1^+(0)},\ S_{41} = \frac{a_2^+(L)}{a_1^+(0)} \quad (13b)$$

The S-parameter results from our coupled-mode theory (CMT) and also those obtained from commercial software ADS Momentum (MoM) are shown in Fig.4.

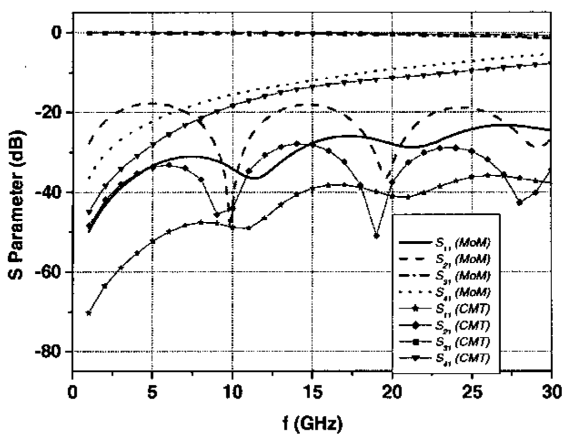

Fig. 4 S Parameters of symmetric coupled microstrip lines

From Fig.4, we can see that for the forward coupling the results from our coupled mode theory agree well with those from ADS Momentum. The frequency dependant characteristics of the backward coupling also agree well, while the absolute values of backward wave calculated by our coupled mode theory is weaker than those by ADS Momentum. This discrepancy is believed to be from the setting of terminations in ADS Momentum (50ohms is used in ADS); while in coupled mode theory, we set the termination to be ideally equal to the characteristic impedance of the individual microstrip line.

The CPU time used by our coupled mode theory for this two coupled microstrip lines is only 5 seconds on a Pentium IV 1.7G personal computer excluding the time of calculating the eigenmodes by the FDTD method. For the same problem, it will take ADS Momentum about 15 minutes to obtain the S-parameters.

V. Conclusion

The non-orthogonal coupled mode theory is extended to the analysis of multi-conductor transmission lines by considering the backward coupling in this paper. This approach can characterize the crosstalk between multi-conductor signal lines in a multilayered structure. Compared to ADS momentum, the coupled mode theory exhibits very high efficiency. This approach can also be applied to more general and complicated problems, such as non-uniform multi-conductor microstrip lines.

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